

# Short Papers

## Coaxial Impedance Inverter Design

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**Abstract**—It is shown that the effects of discontinuity capacitances associated with the abrupt change in the center conductor diameter of a coaxial bandpass filter can readily be anticipated at the designing stage.

Recently Davis and Khan [1] published a paper on "coaxial bandpass filter design," based on an improved design of an impedance inverter with the distributed line lengths of the inverter taken into account. The line elements obtained in this manner can be corrected for discontinuity capacitances by any of several methods.

It will be shown that this can be avoided if the reactive elements resulting from the discontinuity are taken into account from the outset.

We start by calculating the transfer matrix of the circuit shown in Fig. 1, consisting of a line length  $l$  with characteristic impedance  $Z_0$  together with two capacitances  $C_d$  and two lengths  $\Phi/2$  with the characteristic impedance  $Z_c$ , all assumed to be loss-free.

$$\left. \begin{aligned} A = D &= - \left[ B_d Z_0 \cos \Phi + \frac{1}{2} \left( \frac{Z_c}{Z_0} + \frac{Z_0}{Z_c} - B_d^2 Z_0 Z_c \right) \sin \Phi \right] \\ &\quad \cdot \sin \beta l + [\cos \Phi - B_d Z_c \sin \Phi] \cos \beta l \\ j \frac{B}{Z_c} &= \left[ 2 B_d Z_0 \sin \frac{\Phi}{2} \cos \frac{\Phi}{2} - \frac{Z_0}{Z_c} \cos^2 \frac{\Phi}{2} \right. \\ &\quad \left. - \left( B_d^2 Z_0 Z_c - \frac{Z_c}{Z_0} \right) \sin^2 \frac{\Phi}{2} \right] \sin \beta l \\ &\quad + \left[ -2 \sin \frac{\Phi}{2} \cos \frac{\Phi}{2} + 2 B_d Z_c \sin^2 \frac{\Phi}{2} \right] \cos \beta l \\ -j C Z_c &= \left[ -2 B_d Z_0 \sin \frac{\Phi}{2} \cos \frac{\Phi}{2} - \frac{Z_0}{Z_c} \sin^2 \frac{\Phi}{2} \right. \\ &\quad \left. - \left( B_d^2 Z_0 Z_c - \frac{Z_c}{Z_0} \right) \cos^2 \frac{\Phi}{2} \right] \sin \beta l \\ &\quad + \left[ 2 \sin \frac{\Phi}{2} \cos \frac{\Phi}{2} + 2 B_d Z_c \cos^2 \frac{\Phi}{2} \right] \cos \beta l \end{aligned} \right\} \quad (1)$$

Combining the last two equations of (1) gives

$$j \left( \frac{B}{Z_c} - C Z_c \right) = \left( \frac{Z_c}{Z_0} - \frac{Z_0}{Z_c} - B_d^2 Z_0 Z_c \right) \sin \beta l + 2 B_d Z_c \cos \beta l. \quad (2)$$

To obtain the immittance inverting operation at a certain frequency the elements of the above matrix must be chosen so that

$$\left. \begin{aligned} A &= D = 0 \\ B &= jK \\ C &= \frac{j}{K} \end{aligned} \right\} \quad (3)$$

where  $K$  is a real constant.

From the last two equations of (3) it follows that

$$j \left( \frac{B}{Z_c} - C Z_c \right) = \frac{Z_c}{K} - \frac{K}{Z_c}. \quad (4)$$

Equating the terms on the right-hand sides of (2) and (4) we now find

$$\left( \frac{Z_c}{Z_0} - \frac{Z_0}{Z_c} - B_d^2 Z_0 Z_c \right) \sin \beta l + 2 B_d Z_c \cos \beta l = \frac{Z_c}{K} - \frac{K}{Z_c}.$$

Reduced to a quadratic equation of  $\tan \beta l$ , we obtain for the un-

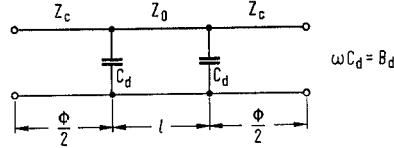


Fig. 1. Equivalent circuit for a disk impedance inverter.

known length  $l$

$$\tan^2 \beta l + \frac{2ab}{a^2 - c^2} \tan \beta l + \frac{b^2 - c^2}{a^2 - c^2} = 0 \quad (5)$$

with

$$\left. \begin{aligned} a &= \frac{Z_c}{Z_0} - \frac{Z_0}{Z_c} - B_d^2 Z_0 Z_c \\ b &= 2 B_d Z_c \\ c &= \frac{Z_c}{K} - \frac{K}{Z_c} \end{aligned} \right\}. \quad (6)$$

Now knowing  $\tan \beta l$ , from the condition  $A = D = 0$  of (3)  $\tan \Phi$  may be found from (1)

$$\tan \Phi = \frac{1 - B_d Z_0 \tan \beta l}{B_d Z_c + \frac{1}{2} \left( \frac{Z_0}{Z_c} + \frac{Z_c}{Z_0} - B_d^2 Z_0 Z_c \right) \tan \beta l}. \quad (7)$$

## REFERENCES

[1] W. A. Davis and P. J. Khan, "Coaxial bandpass filter design," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 373-380, Apr. 1971.

## Comments on "A Relationship Between the Scattering Parameters of a Passive Lossy Nonreciprocal Two-Port"

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**Abstract**—An inequality relating the scattering coefficients of a passive lossy-reciprocal or nonreciprocal two-port is derived. For the reciprocal two-port the inequality reduces to that presented by Uhlir.

## INTRODUCTION

In this letter we derive an upper-bound inequality that applies to the scattering coefficients of a passive lossy nonreciprocal two-port. The inequality given here corrects the inequality derived in the above correspondence,<sup>1</sup> and reduces to the inequality given by Uhlir [1] for the reciprocal case. This inequality allows one to bound the magnitude of the reflection coefficient of one port using measurements of the magnitudes of the reflection coefficient of the other port and the insertion losses of the network in both directions. Such a bound simplifies the automated testing of microwave two-ports.

Let the elements of the scattering matrix of a two-port be given by  $S_{11}$ ,  $S_{12}$ ,  $S_{21}$ ,  $S_{22}$  and their phases be given by  $\theta_{11}$ ,  $\theta_{12}$ ,  $\theta_{21}$ , and  $\theta_{22}$ , respectively.

Equation (3) of footnote one is given by:

$$(1 - |S_{11}|^2 - |S_{21}|^2)(1 - |S_{22}|^2 - |S_{12}|^2) \geq |S_{11}^* S_{12} + S_{21}^* S_{22}|^2 \quad (1)$$

where  $| |$  denotes magnitude and  $S^*$  denotes the complex conjugate of  $S$ . After putting only those terms that contain  $|S_{22}|$  on the right-

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<sup>1</sup> H. J. Hindin, *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-19, p. 781, Sept. 1971.